FAKULTÄT FÜR INFORMATIK

DER TECHNISCHEN UNIVERSITÄT MÜNCHEN

Probabilistic Cellular Automata

Carlos Camino

Outline of the Presentation

1. Cellular Automata
2. Majority Problem
3. Tool
4. Analysis
5. Results
6. Conclusions

1. Cellular Automata

1. Cellular Automata

Cells ($n = 10$)

0	1	2	3	4	5	6	7	8	9

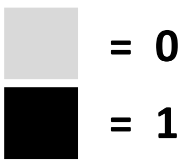
1. Cellular Automata

0	1	2	3	4	5	6	7	8	9

Neighborhood ($r = 1$)

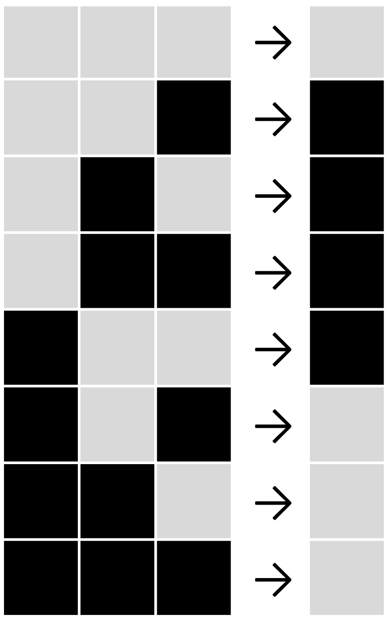
1. Cellular Automata

States ($s = 2$)



1. Cellular Automata

Rule



1. Cellular Automata

Configuration

t = 0

t = 1

t = 2

t = 3

t = 4

t = 5

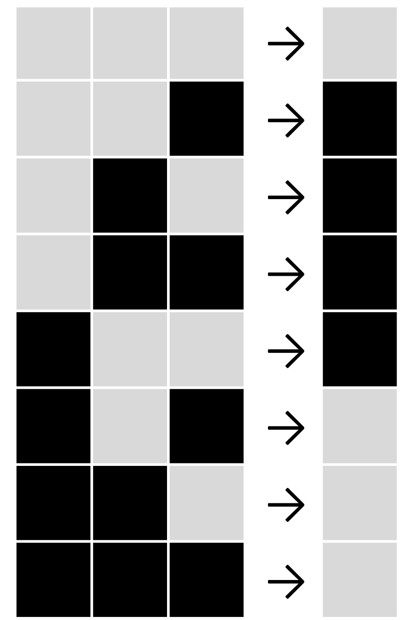
t = 6

t = 7

t = 8

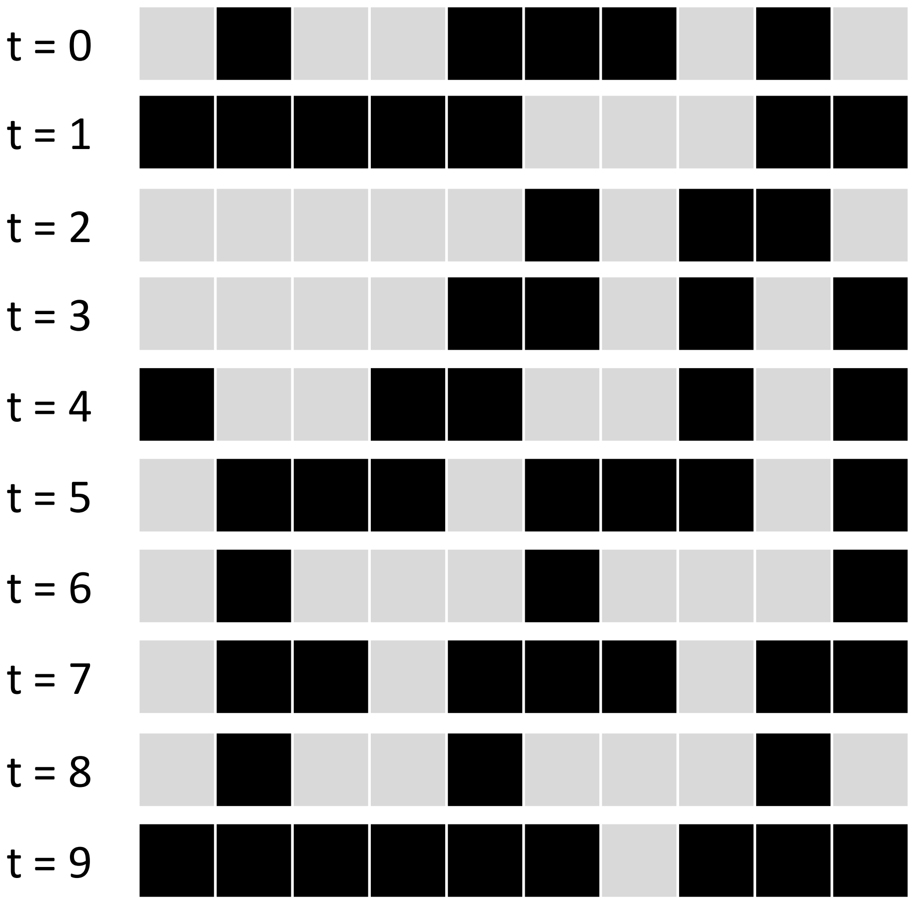
t = 9

Rule

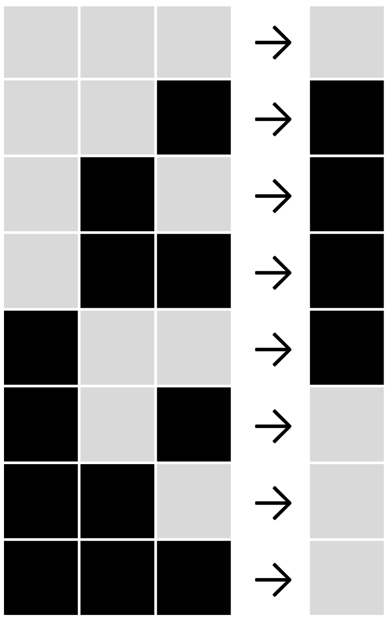


1. Cellular Automata

Configuration



Rule



2. Majority Problem

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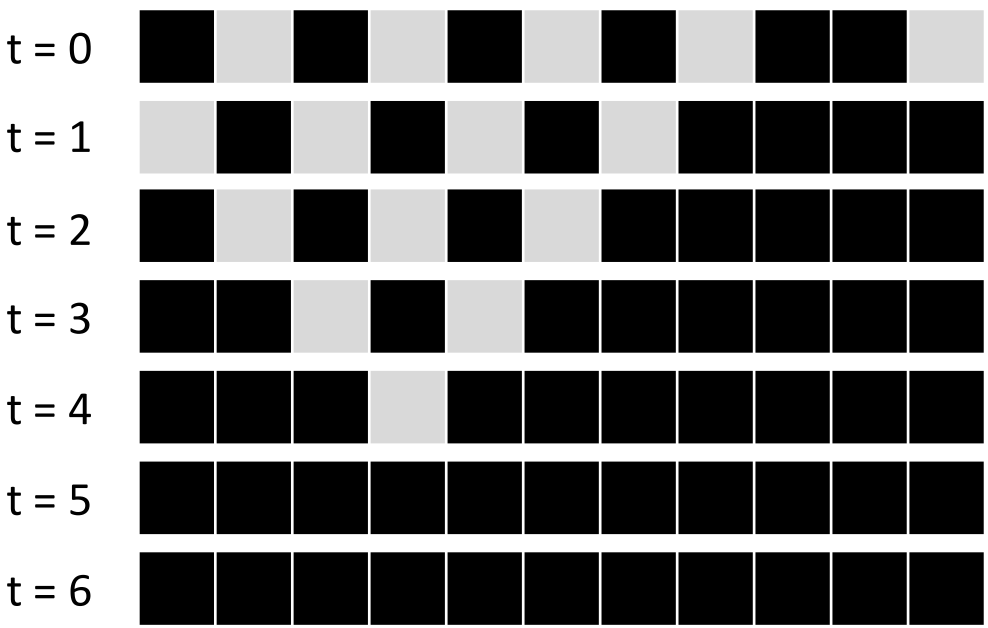
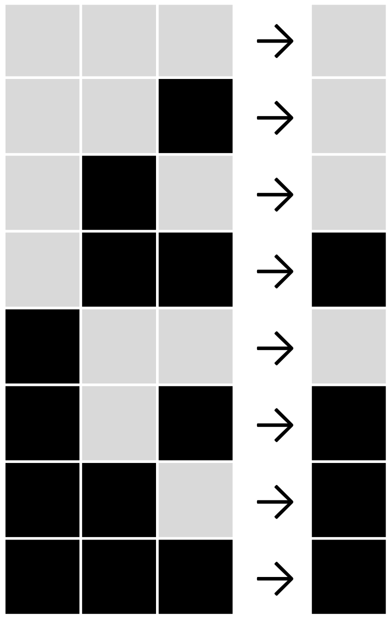
Example 1: solved

Configuration

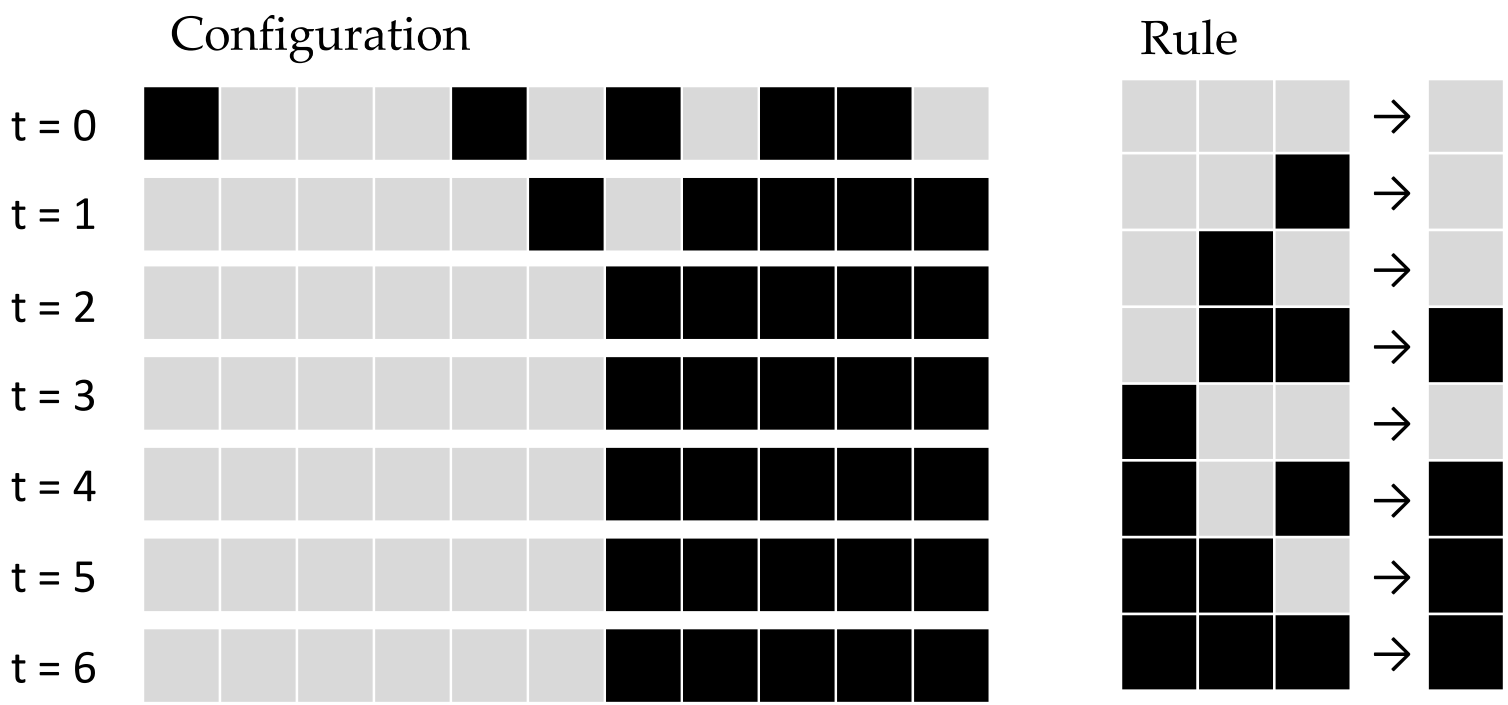


Rule



2. Majority Problem

Example 2: unsolved



2. Majority Problem

Some important solutions for $r = 3$ and $n = 149$:

Year	Authors	Method	Performance
1978	Gács, Kurdyumov, Levin	human-writen	81.6%
1994	Das, Mitchell, Crutchfield	Genetic Algorithm	76.9%
1995	Davis	human-writen	81.8%
1995	Das	human-writen	82.178%
1996	Andre, Bennett, Koza	Genetic Programming	82.326%
1998	Juillé, Pollack	Coevulotionary Learning	86.3%

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Question: Does a perfect rule exist?

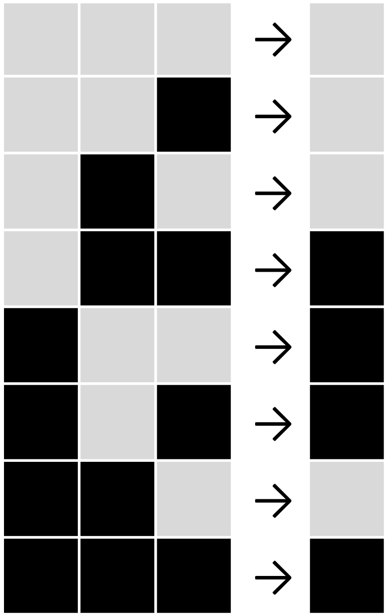
2. Majority Problem

Variation 1: (Capcarre, Sipper and Tomassini - 1996)

Change the output specification:

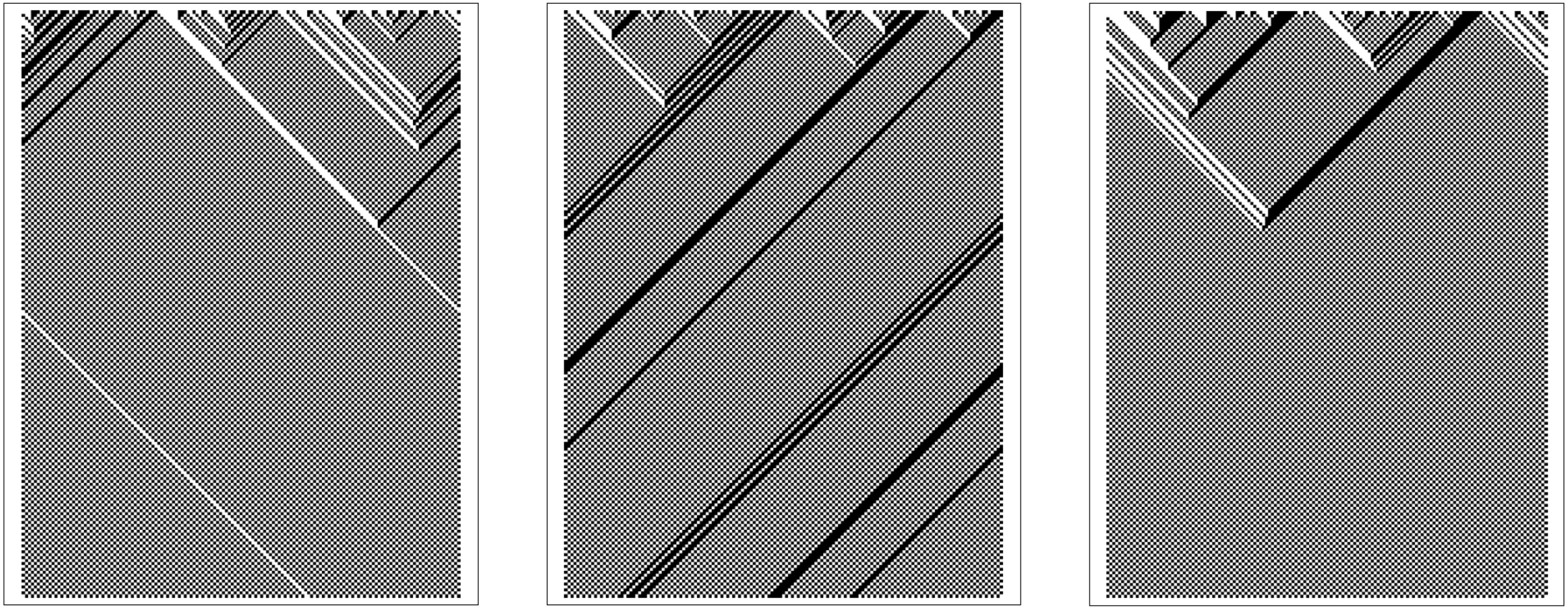
If the initial configuration contains more 1's (or 0's) than 0's (or 1's), no two cells with state 0 (or 1) can coexist in the final configuration.

Rule 184



2. Majority Problem

Variation 1: (Capcarre, Sipper and Tomassini - 1996)



Source: Mathieu S. Capcarrere, Moshe Sipper, and Marco Tomassini. Two-state, r = 1 Cellular Automaton that classifies density. *Physical Review Letter*, 77 (24):4969-4971, 1996.

2. Majority Problem

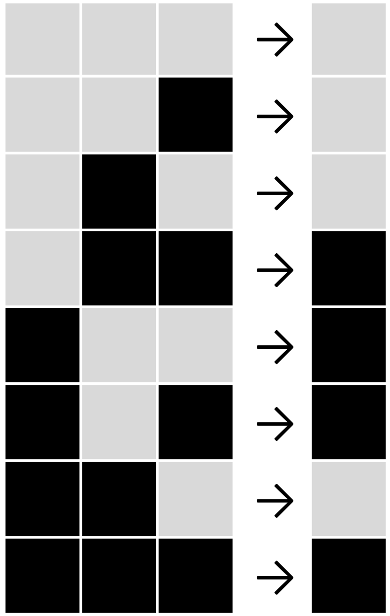
Variation 2: (Fuks - 1997)

Use two Cellular Automata:

Combine the use of Rule 184 and Rule 232.

First apply only Rule 184, then only Rule 232.

Rule 184



2. Majority Problem

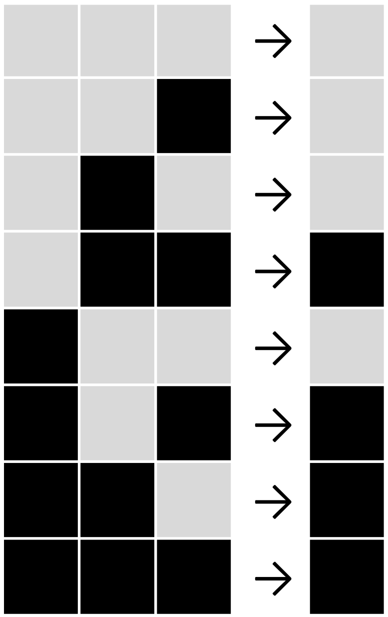
Variation 2: (Fuks - 1997)

Use two Cellular Automata:

Combine the use of Rule 184 and Rule 232.

First apply only Rule 184, then only Rule 232.

Rule 232

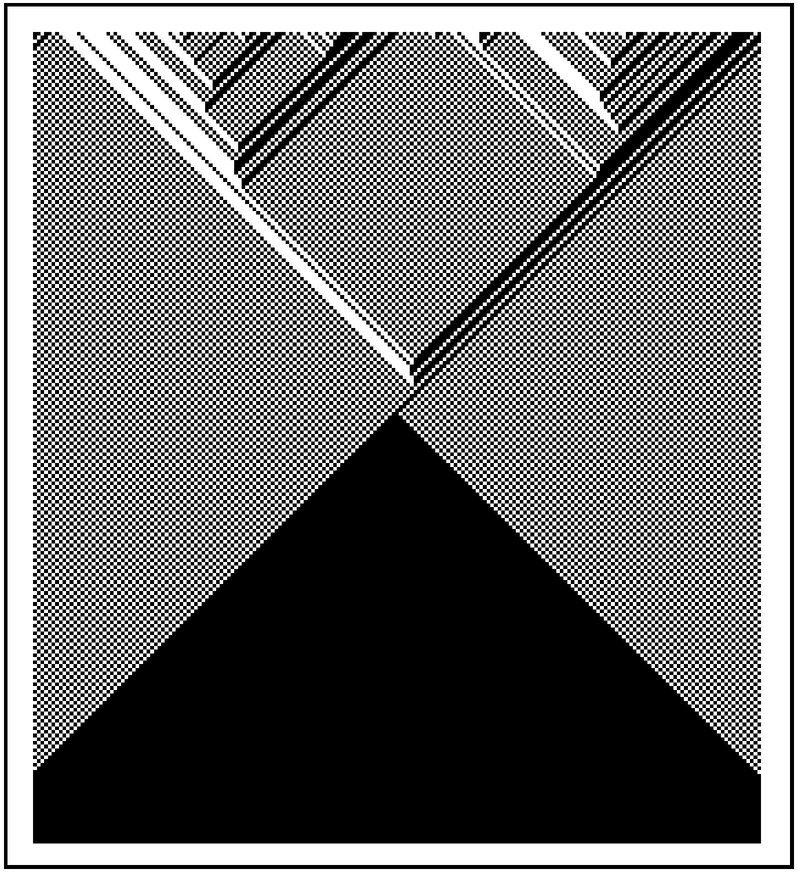


2. Majority Problem

Variation 2: (Fuks - 1997)

Rule 184

Rule 232



3. Tool

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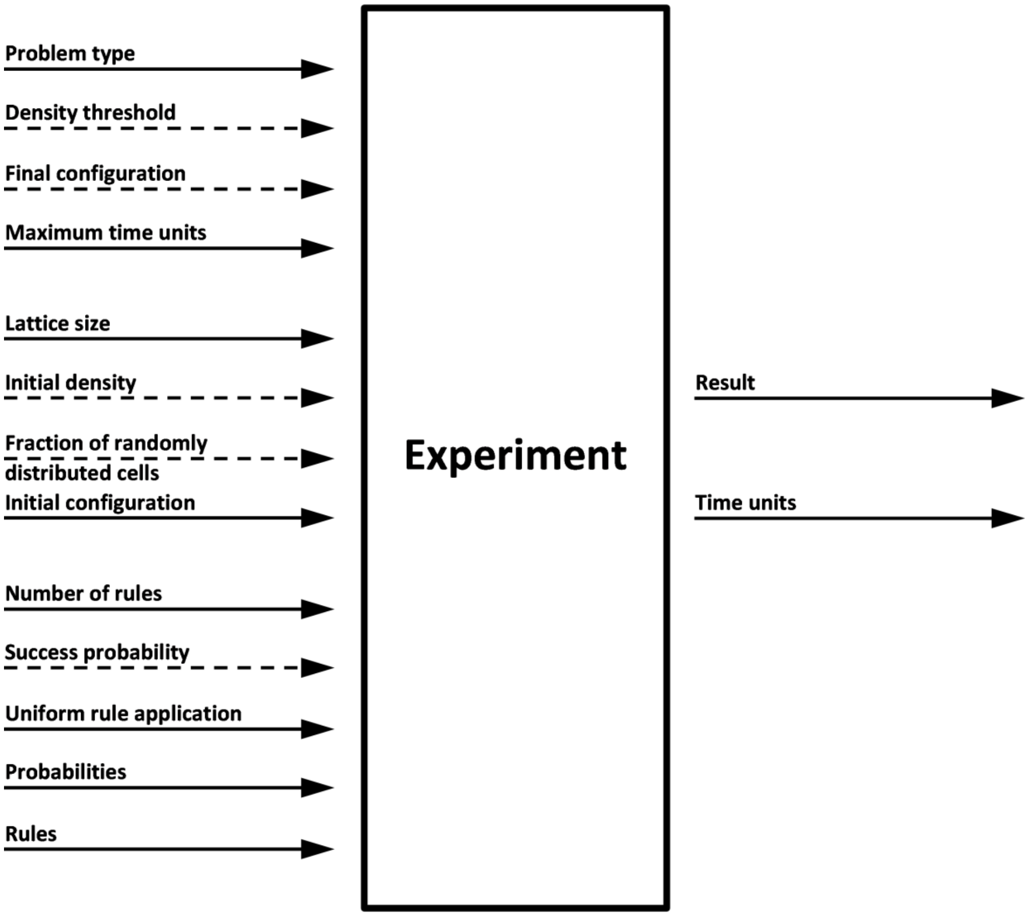
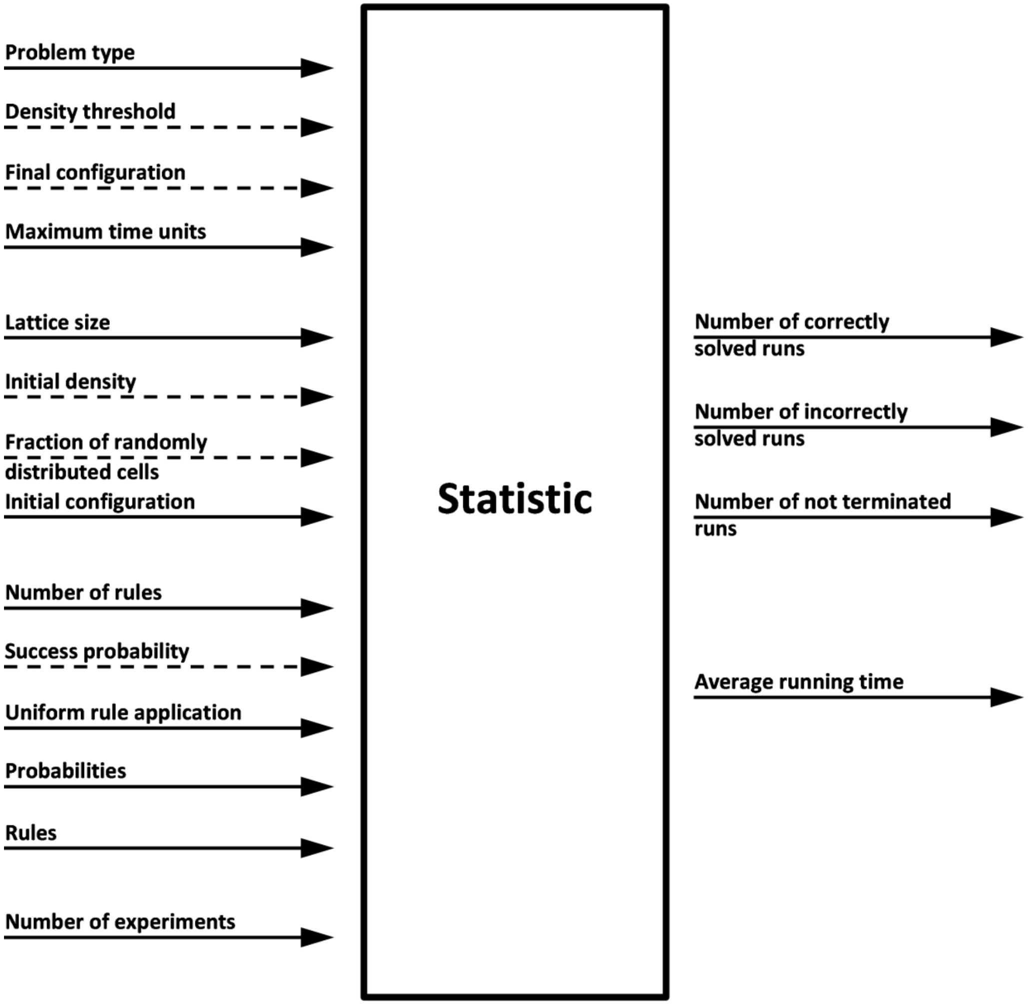
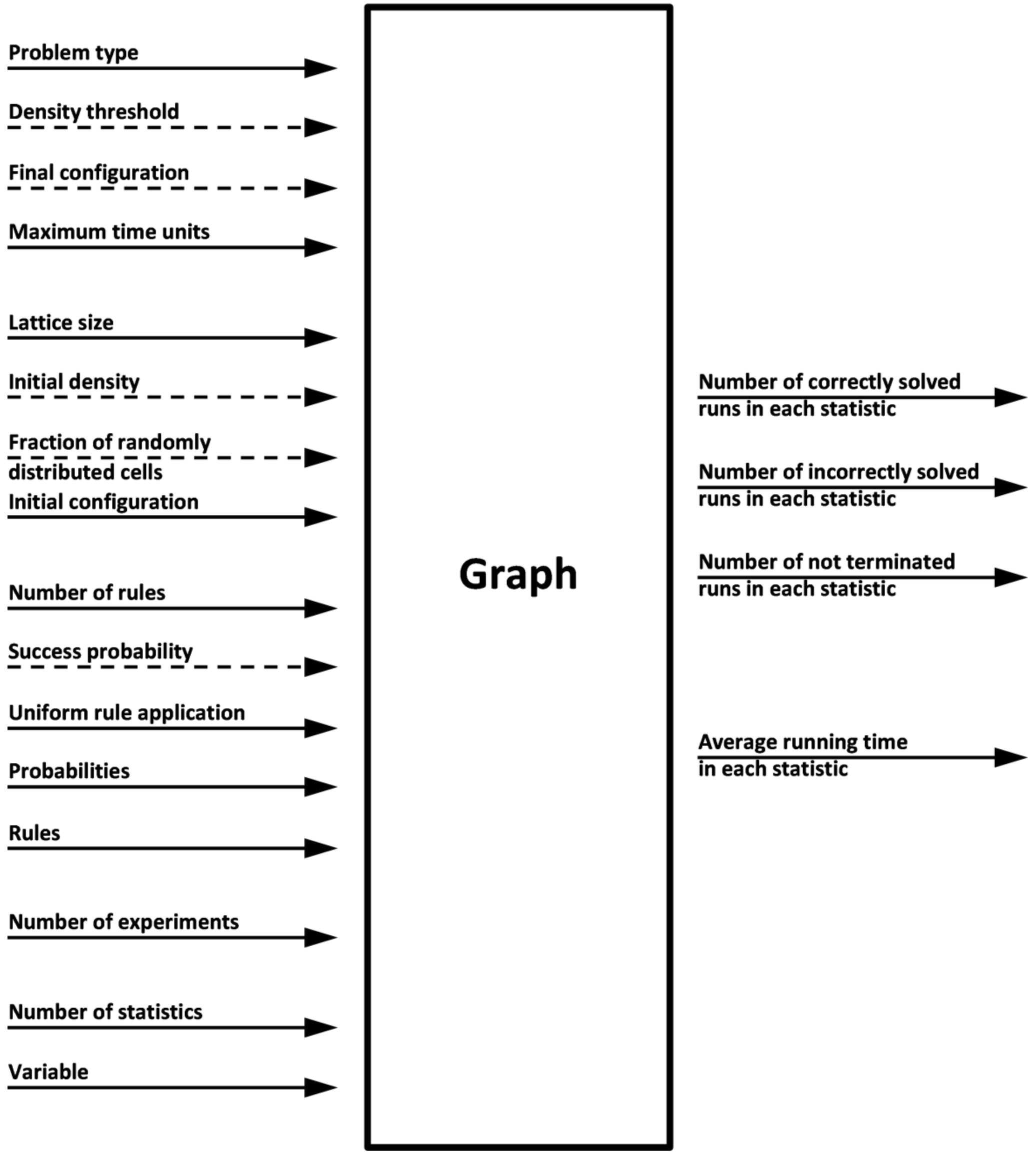

Problem type
Density threshold
Final configuration
Maximum time units
Lattice size
Initial density
Fraction of randomly distributed cells
Initial configuration
Number of rules
Success probability
Uniform rule application
Probabilities
Rules
Experiment
Result
Time units

3. Tool



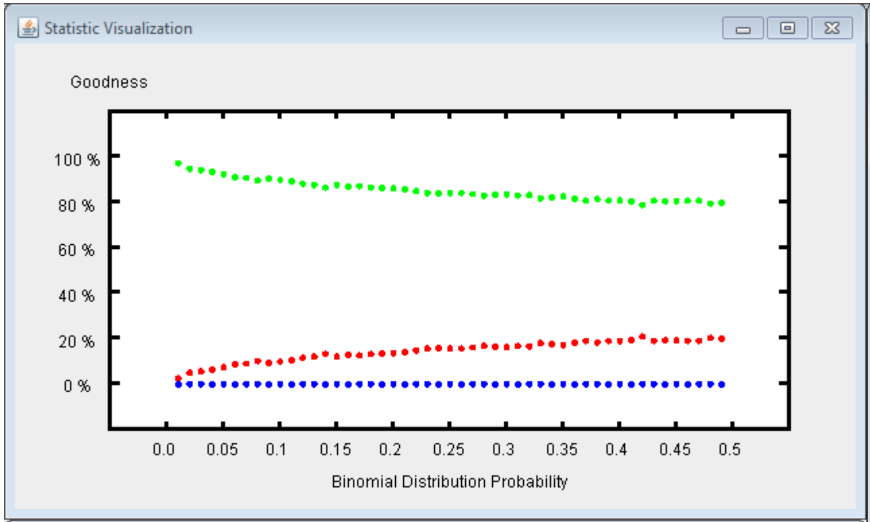
Problem type
Density threshold
Final configuration
Maximum time units
Lattice size
Initial density
Fraction of randomly distributed cells
Initial configuration
Number of rules
Success probability
Uniform rule application
Probabilities
Rules
Number of experiments
Statistic
Number of correctly solved runs
Number of incorrectly solved runs
Number of not terminated runs
Average running time

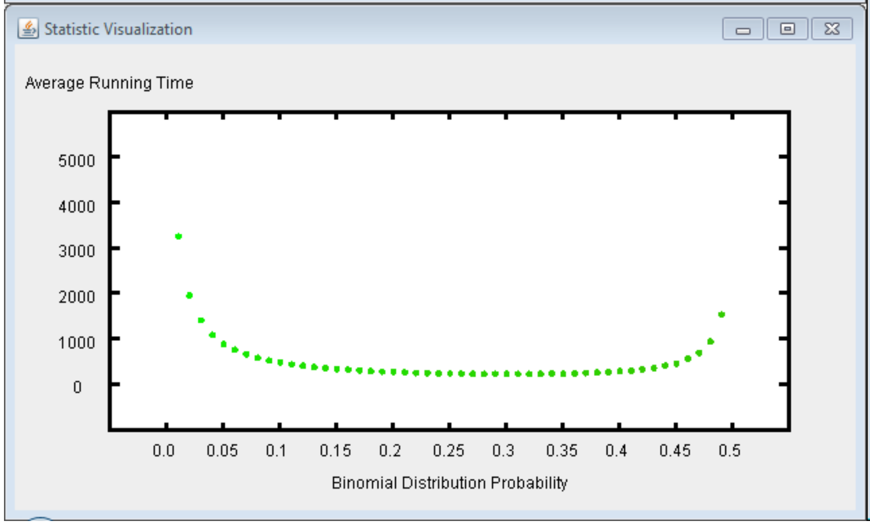
3. Tool

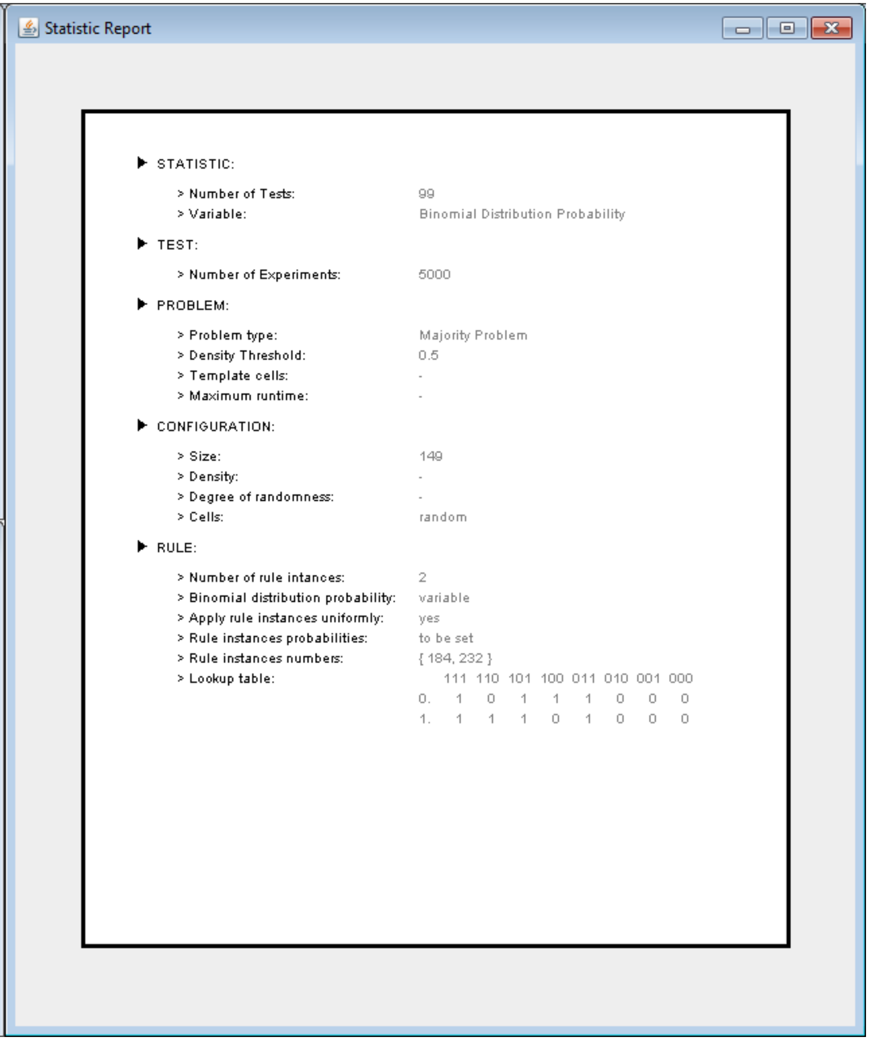

Problem type
Density threshold
Final configuration
Maximum time units
Lattice size
Initial density
Fraction of randomly distributed cells
Initial configuration
Number of rules
Success probability
Uniform rule application
Probabilities
Rules
Number of experiments
Number of statistics
Variable
Graph
Number of correctly solved runs in each statistic
Number of incorrectly solved runs in each statistic
Number of not terminated runs in each statistic
Average running time in each statistic

4. Analysis

4. Analysis







Statistic Report

▸ STATISTIC:

- > Number of Tests: 99
- > Variable: Binomial Distribution Probability

▸ TEST:

- > Number of Experiments: 5000

▸ PROBLEM:

- > Problem type: Majority Problem
- > Density Threshold: 0.5
- > Template cells: -
- > Maximum runtime: -

▸ CONFIGURATION:

- > Size: 149
- > Density: -
- > Degree of randomness: -
- > Cells: random

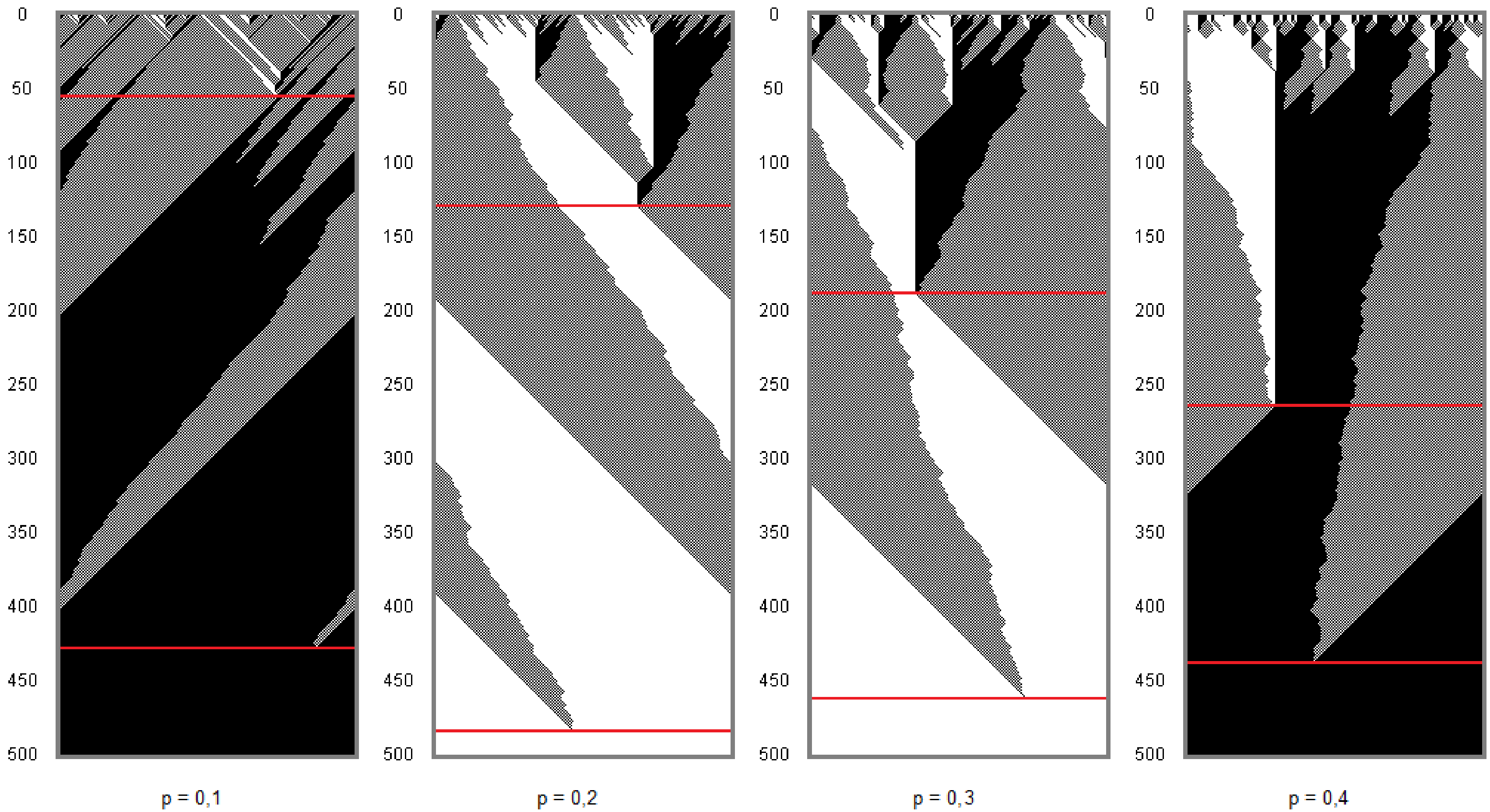
▸ RULE:

- > Number of rule intances: 2
- > Binomial distribution probability: variable
- > Apply rule instances uniformly: yes
- > Rule instances probabilities: to be set
- > Rule instances numbers: { 184, 232 }
- > Lookup table:

	111	110	101	100	011	010	001	000
0.	1	0	1	1	1	0	0	0
1.	1	1	1	0	1	0	0	0

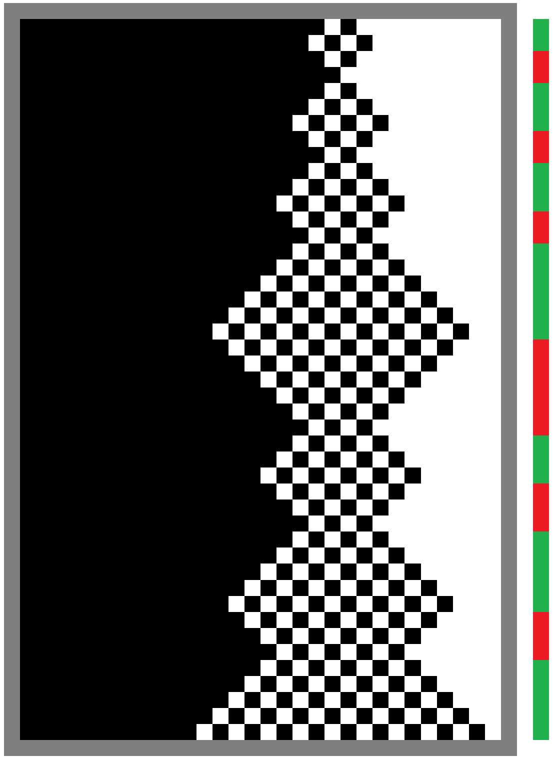
4. Analysis

Total running time as the sum of the running time of two phases:



4. Analysis

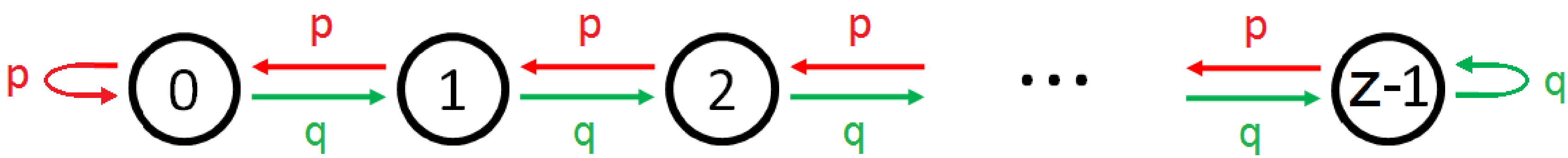
Phase 1 simulated with n = 30 , d = 2/3 and p = 0.4 :



Green: Rule 184

Red: Rule 232

Phase 1 modelled as a random-walker



(q = 1–p)

with stochastic matrix :

$$P = \begin{pmatrix} p & q & 0 & 0 & 0 & \cdots & 0 & 0 \\ p & 0 & q & 0 & 0 & \cdots & 0 & 0 \\ 0 & p & 0 & q & 0 & \cdots & 0 & 0 \\ 0 & 0 & p & 0 & q & \cdots & 0 & 0 \\ p & q & 0 & p & 0 & \ddots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \ddots & q & 0 \\ 0 & 0 & 0 & 0 & \cdots & p & 0 & q \\ 0 & 0 & 0 & 0 & \cdots & 0 & p & q \end{pmatrix}$$

4. Analysis

Expected number of steps from state 0 to state z-1 for some z :

$$h_{0,z-1} = \begin{cases} 0, & \text{if } z = 1, \\ 1q^{-1}, & \text{if } z = 2, \\ 1q^{-1} + 1q^{-2}, & \text{if } z = 3, \\ 2q^{-1} + 0q^{-2} + 1q^{-3}, & \text{if } z = 4, \\ 2q^{-1} + 2q^{-2} - 1q^{-3} + 1q^{-4}, & \text{if } z = 5, \\ 3q^{-1} + 0q^{-2} + 3q^{-3} - 2q^{-4} + 1q^{-5}, & \text{if } z = 6, \\ 3q^{-1} + 3q^{-2} - 3q^{-3} + 5q^{-4} - 3q^{-5} + 1q^{-6}, & \text{if } z = 7, \\ 4q^{-1} + 0q^{-2} + 6q^{-3} - 8q^{-4} + 8q^{-5} - 4q^{-6} + 1q^{-7}, & \text{if } z = 8, \\ \vdots & \end{cases}$$

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Expected number of steps from state 0 to state z-1 for some z :

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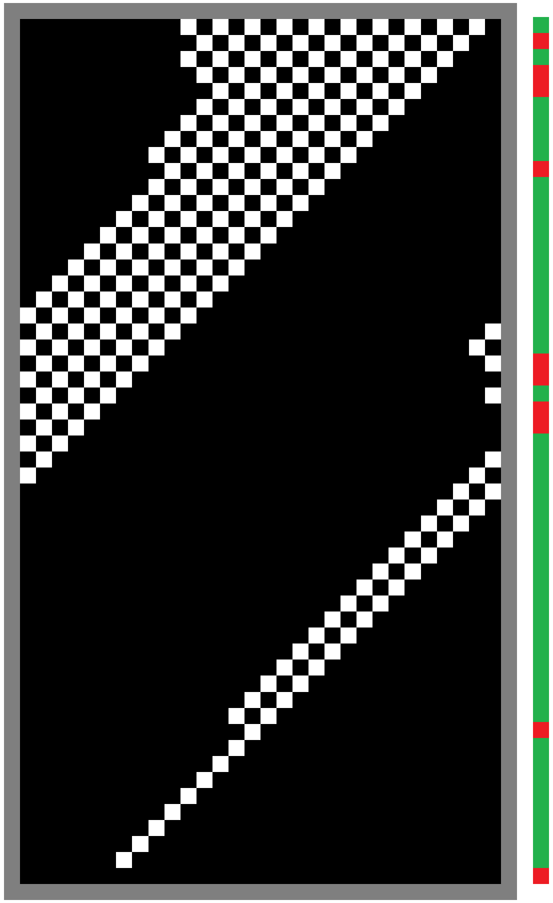
And as a recursion :

$$h_{0,0} = 0,$$
$$h_{0,z} = (h_{0,z-1} + z)q^{-1} - h_{0,z-1}$$

4. Analysis

Phase 2 simulated with n = 30 , d = 2/3 and p = 0.3 :



Green: Rule 184

Red: Rule 232

5. Results

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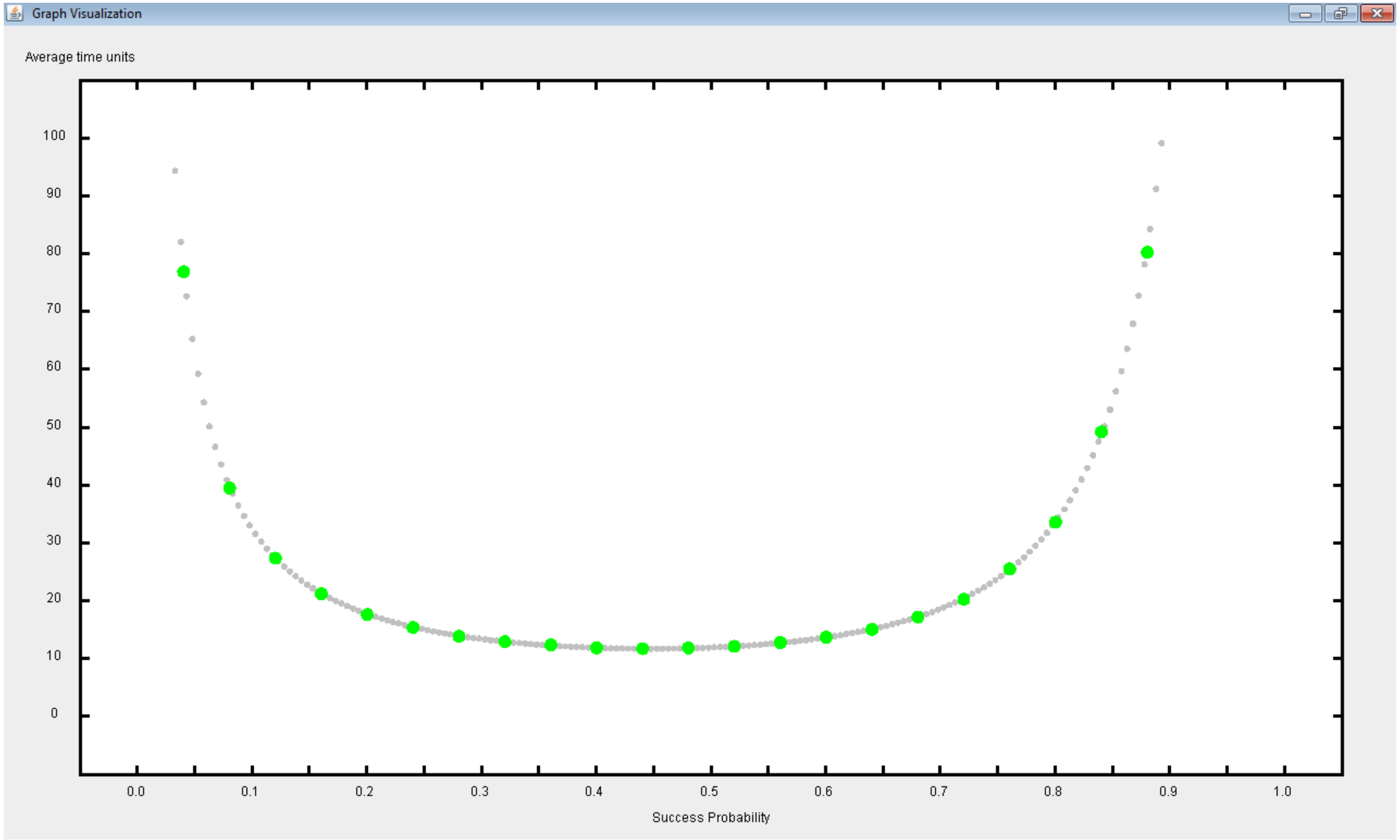
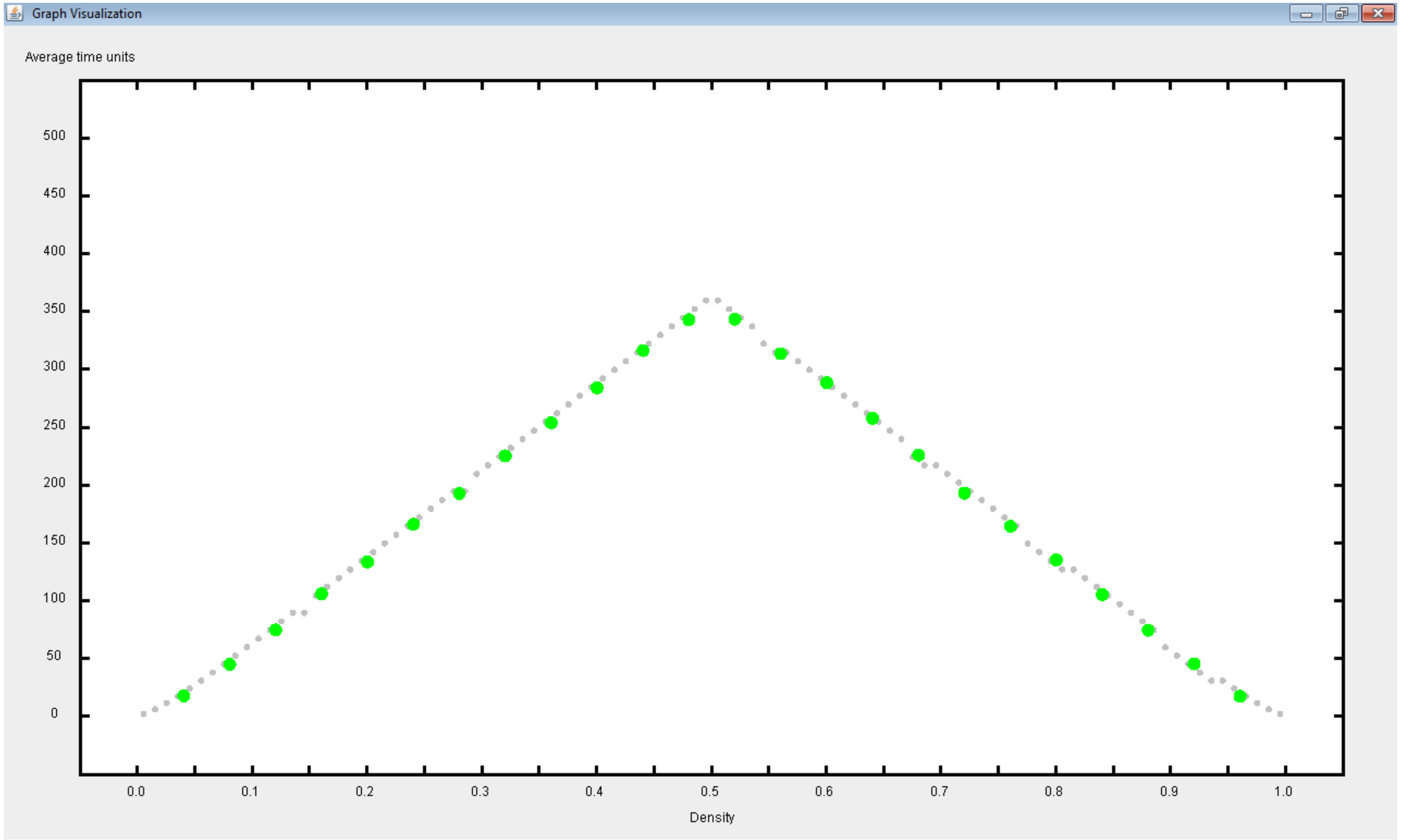


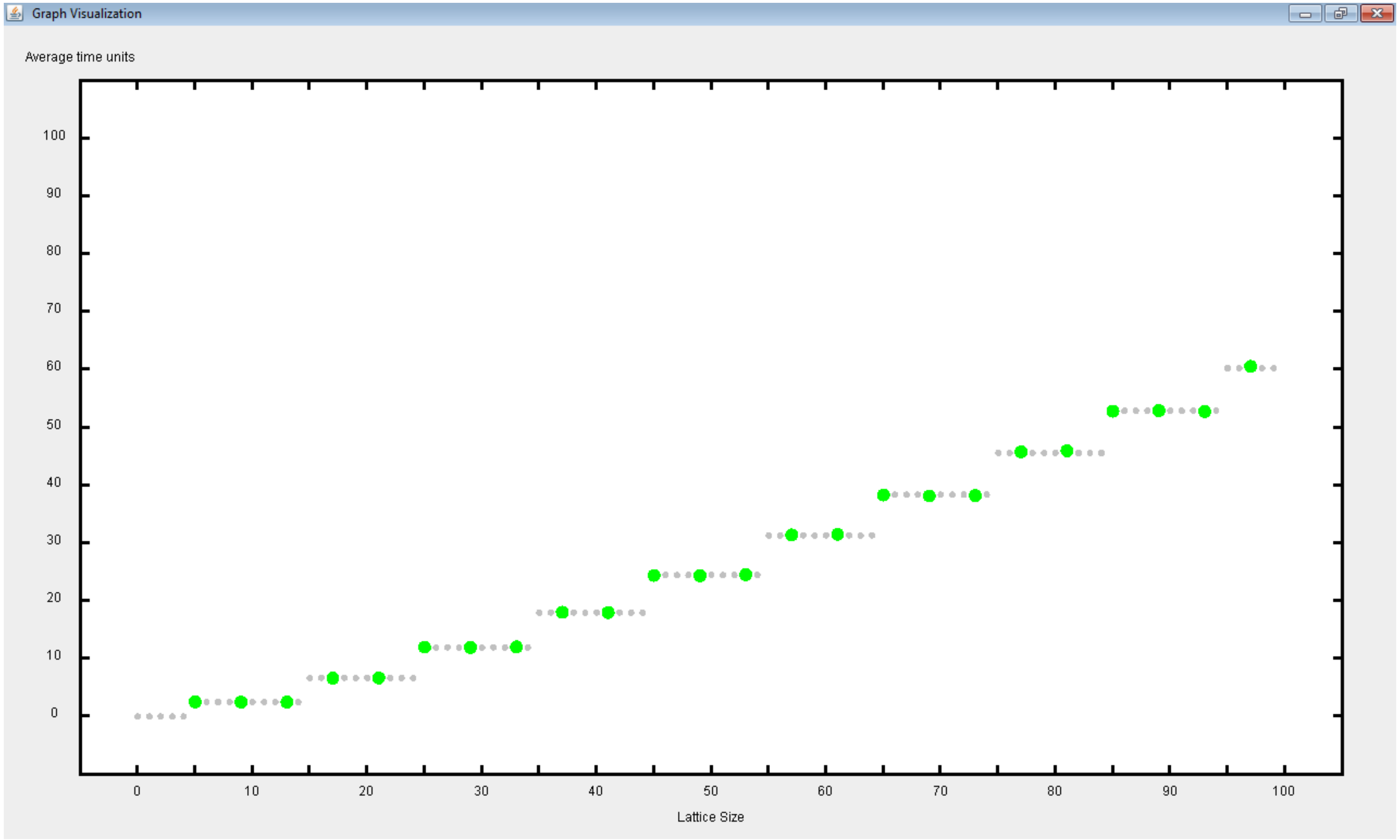
Running Time against p with $d = 0.3$ and $n = 10$

5. Results



Running Time against d with p = 0.4 and n = 100

5. Results



Running Time against n with $d = 0.1$ and $p = 0.4$

6. Conclusions

Expected time

$$
\begin{aligned}
\mathbb{E}[T] &= \mathbb{E}[T_1 + T_2] \\
&=
\end{aligned}
$$

Expected time

$$
\begin{aligned}
\mathbb{E}[T] &= \mathbb{E}[T_1 + T_2] \\
&= \mathbb{E}[T_1] + \mathbb{E}[T_2] \\
\\
&=
\end{aligned}
$$

Expected time

$$
\begin{aligned}
\mathbb{E}[T] &= \mathbb{E}[T_1 + T_2] \\
&= \mathbb{E}[T_1] + \mathbb{E}[T_2] \\
&= \frac{p((\frac{p}{1-p})^{z-1} - z - 2) + (1-p)(z-1)}{(1-2p)^2} +
\end{aligned}
$$

Expected time

$$
\begin{aligned}
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&= \mathbb{E}[T_1] + \mathbb{E}[T_2] \\
&= \frac{p((\frac{p}{1-p})^{z-1} - z - 2) + (1-p)(z-1)}{(1-2p)^2} + \frac{z}{p}
\end{aligned}
$$

Expected time

$$
\begin{aligned}
\mathbb{E}[T] &= \mathbb{E}[T_1 + T_2] \\
&= \mathbb{E}[T_1] + \mathbb{E}[T_2] \\
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\end{aligned}
$$